Radius of curvature

Basic gear formulas:

\[
D_p = \frac{Z}{d_0}
\]

\[
d_b = d_0 \times \cos \phi \quad [\text{in.}]
\]

\[
t = \frac{\pi}{2D_p} \quad [\text{in.}]
\]

Conversion from degrees to radians:

\[
\hat{\phi} = \frac{\pi \phi}{180} \quad [\text{rad.}]
\]

Where:
Z – number of teeth
\(d_0\) – pitch diameter [in.]
\(d_b\) – base diameter [in.]
\(D_p\) – diametral pitch [dimensionless]
\(\phi\) – pressure angle [degree]
t – CTT, circular tooth thickness [in.]

General form of an involute function:

\[
\hat{\theta} = \text{inv}\phi = \tan \phi - \hat{\phi}
\]

Where:
\(\hat{\theta}\) – polar angle [radians]
\(\theta\) – polar angle [degree]
inv\(\phi\) – involute function of an angle [rad.]
\(\phi\) – involute pressure angle [degree]
\(\hat{\phi}\) – involute pressure angle [radians]

Radius of curvature at an arbitrary point on the involute curve:

\[
\rho_A = (R_A^2 - R_b^2)^{0.5}
\]

Where:
\(\rho_A\) – radius of curvature, at point "A" on the involute
\(R_A\) – radius to point "A"
\(R_b\) – base radius

This could be expressed as a function of the pressure angle:

\[
\rho_A = R_A \times \sin \phi_A
\]

Where:
\(\phi_A\) – pressure angle at point "A"
Radius of curvature at pitch diameter:

\[ \rho = \frac{d \cdot \sin \phi}{2} \]

Where:  
- \( \rho \) – radius of curvature  
- \( d \) – pitch diameter  
- \( \phi \) – pressure angle at pitch diameter

NOTE:

As \( R_b \to \infty \) the \( \rho \to \infty \) and therefore the tooth shape becomes a straight line as in the basic involute rack
Involute

\[ \theta = \text{inv} \Phi_A \]

Radius of curvature \( \rho \) at point \( A \)

Base radius \( R_b \)

\[ \Phi_A \]